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COMMENT

Self-avoiding walks on diluted lattices near the percolation threshold

Yup Kim†
Department of Physics, Purdue University, West Lafayette, IN 47907, USA

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Abstract. Using the node-link picture of the infinite cluster near the percolation threshold, a crossover of the critical behaviour of the self-avoiding walks on the random lattice near the percolation threshold (or just above threshold) is suggested and a scaling function of this crossover is proposed. We compare the result of our qualitative argument with a Monte Carlo simulation result.

Recently there has been considerable interest (Hiley *et al* 1977, Chakrabarti and Kertesz 1981, Kremer 1981, Harris 1983a, Kim 1983, Rammal *et al* 1984, Kim and Khang 1985) in self-avoiding walks (SAW) on the disordered lattice. This problem is a physically valuable model for the statistics of linear polymers on the random medium. The SAW consists of $(N+1)$ monomers connected by N bonds of equal length. The main interest is the mean square radius of $\langle R^2\rangle$ with well known results (de Gennes 1976)

$$\langle R^2\rangle \sim N^{2\nu} \tag{1}$$

where the exponent ν is a universal quantity in each dimension d. Recently Chakrabarti and Kertesz (1981) argued that the exponent ν is equal to $\frac{1}{2}$ on the disordered lattice regardless of dimension. However, for the weakly diluted lattice, by which we mean the random lattice far above the percolation threshold (Stauffer 1979, Essam 1980), Harris (1983a) has modified his criterion (Harris 1974) and has shown that ν is the same as that in the undiluted case. Using a field-theoretic argument and the renormalisation group ε expansion Kim (1983) has also shown that the exponent on the weakly diluted lattice is the same as that on the non-random lattice. Kremer (1981) has done a simulation on the random diamond lattice and has found that on the weakly diluted lattice the exponent is the same as that in the non-random case. On the other hand, just above the percolation threshold he has found a crossover of the exponent ν to a new higher one. In contrast to the result of Kremer's simulation Rammal *et al* (1984) have shown that the exponent ν of SAW on the fractal lattices (Mandelbrot 1982) is smaller than that on the non-random lattice. If fractal lattices represent the backbone of the infinite cluster just above the percolation threshold (Gefen *et al* 1981), there is a contradiction between the analytical study on fractals and a numerical simulation. Although SAW on disordered lattices have been extensively studied, this somewhat contradictory problem has not yet been explained clearly. We therefore want to suggest a crossover model of SAW on random lattices just above the percolation threshold.

† Present address: Department of Physics, Research Institute for Basic Sciences, Kyung Hee University, Seoul 131, South Korea

Our model is based on the qualitative, but physically plausible, picture for the backbone of the infinite cluster near percolation threshold. It is the purpose of this comment to study a physical origin of the crossover in order to invoke more exact studies on this subject.

A very useful picture of a randomly diluted lattice which has been proposed by de Gennes (1976) and Skal and Shklovskii (1975) is as follows: for the diluted lattice just above the percolation threshold the lattice can be viewed as a collection of nodes which are connected by links which can be thought of as random paths (see figure 1). Two important lengths enter this picture. One is the distance between nodes which is of the order of the percolation correlation length

$$\xi_{\mathrm{p}} \sim (p-p_{\mathrm{c}})^{-\nu_{\mathrm{p}}} \tag{2}$$

where p is the concentration of the occupied bonds (sites) of the diluted lattice, p_{c} is the critical concentration of the percolation transition (Stauffer 1979) and ν_{p} is the correlation length exponent of the percolation transition. The other important length is the length L of the random path between nodes which is given by the new exponent ζ through

$$L \sim (p-p_{\mathrm{c}})^{-\zeta}. \tag{3}$$

Just above the threshold the backbone of the infinite cluster is built up by the anomalous (or 'fractal') region and normal region (Gefen *et al* 1981). If the number of steps N of SAW is larger than L (or $N > L$), a dilute lattice would behave as a normal d-dimensional lattice. We call this the 'normal' regime. In normal regime, $\langle R^2 \rangle$ is still given by (Flory 1969)

$$\langle R^2 \rangle \sim N^{2\nu_{\mathrm{n}}} \tag{4}$$

where ν_{n} is the same as the exponent in the non-random lattice which is given by

$$\nu_{\mathrm{n}} = 3/(2+d) \tag{5}$$

in d dimensions. If $N < L$, the diluted lattice would behave anomalously like a fractal and we call this regime the anomalous regime. In anomalous regime, $\langle R^2 \rangle$ should be given by

$$\langle R^2 \rangle \sim N^{2\nu_{\mathrm{a}}} \tag{6}$$

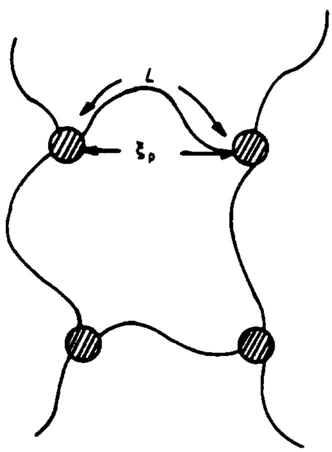

Figure 1. Node-link picture of the infinite cluster just above percolation threshold.

where ν_a is the new 'anomalous' exponent. The scaling form for $\langle R^2 \rangle$ should be

$$\langle R^2 \rangle^{1/2} \sim N^{\nu_a} f[N(p-p_c)^{\zeta}] \tag{7}$$

if we consider the crossover around $N \sim L$. Here $f(x)$ satisfies

$$f(x) \to \text{constant} \qquad x \to 0 \tag{8a}$$

and

$$f(x) \to x^{\nu_n} - x^{\nu_a} \qquad x \to \infty. \tag{8b}$$

One possible formula for ν_a is

$$\nu_a = \nu_f = \frac{1}{d_B} \frac{3\bar{d}_B}{\bar{d}_B + 2} \tag{9}$$

which is suggested by Rammal *et al* (1984) based on the explicit calculation on the special fractals. Here d_B is the fractal dimension (Mandelbrot 1982) of the backbone of the infinite cluster and $\bar{d}_B$ is the spectral dimension (Rammal *et al* 1984) of the backbone. The fractal exponent ν_f based on (9) is smaller than ν_n in the normal regime. Another possible formula is

$$\nu_a = \nu_q \sim \nu_p / \zeta \tag{10}$$

which is from the geometrical consideration for the node-link picture of the backbone of the infinite cluster (see figure 1). The reason for (10) is as follows. If $N < L$,

$$\langle R^2 \rangle \sim \xi_p^2 \tag{11}$$

as can be seen from figure 1. Then from (2) and (3),

$$\langle R^2 \rangle \sim N^{2\nu_p/\zeta}. \tag{12}$$

Using the numerical data for the various exponents (Harris 1983b) of percolation transition, we calculate the exponent ν_q in each dimension. The results for dimensions between 3 and 6 are displayed in table 1. We also display ν_f (Rammal *et al* 1984) and ν_n for comparison. As is seen from table 1, the upper critical dimension of the crossover phenomena should be 6, because the exponent ν of SAW in each regime is $\frac{1}{2}$ in six dimensions. In the normal regime the upper critical dimension is 4, because the critical phenomena of SAW in this regime is in the same universality class as that of the $n=0$ n-vector model of magnetism (de Gennes 1979). However, in anomalous regime the upper critical dimension should be 6, because (ν_q or ν_f) is $\frac{1}{2}$ in six dimensions. If one succeeds in the exact formulation of the Landau-Ginzburg functional of SAW on the random lattice for renormalisation group ε expansion for the dependable study

Table 1. Exponents for self-avoiding walks near percolation threshold.

Dimension	3	4	5	6
ν_p	0.85 ± 0.02	0.66 ± 0.02	0.51 ± 0.02	$\frac{1}{2}$
ζ	1.22 ± 0.66	1.05 ± 0.04	1.02	1
ν_n	0.6	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
ν_q	0.70 ± 0.03	0.63	0.56	$\frac{1}{2}$
ν_f	0.57 ± 0.02	0.49 ± 0.03	?	$\frac{1}{2}$

ν_p and ζ are taken from Harris (1983b).
ν_n is from Flory's formula (see (5)).
ν_q is from (10).
ν_f is from Rammal *et al* (1984).

of the crossover phenomena there might be a cubic term in the functional similar to the cubic term in the simple percolation transition (Harris *et al* 1975) and this term, together with the normal functional of the $n=0$ n-vector model, might be able to explain the crossover phenomena near the percolation transition. This argument might be wrong, but it is very suggestive for the field-theoretic approach to SAW on the disordered lattice.

Using the similar scaling form to (7) and his simulation data Kremer estimated the crossover exponent in three dimensions which is nearly equal to our crossover exponent ζ. His new exponent ν_{p_c} which should be corresponding to ν_a in (6) is 0.65 and is very close to our ν_q which is derived from the node-link picture. Both Kremer's simulation result and our qualitative argument based on the node-link picture predict that the new exponent (i.e. ν_q) is larger than that in the normal regime (ν_n). In contrast the exponent ν_f which has been derived by some renormalisation group method on the specific fractals is smaller than ν_n in each dimension. However, in fractals two lattice points are multiply connected and thus ν_f should be smaller than ν_q, because in the node-link picture two lattice points are essentially simply connected. As was explained by Coniglio (1982) the fractal picture for the backbone of the infinite cluster gives good results in low dimensions, while the node-link picture fails to represent the cluster structure for the low dimensionality. Kremer's simulation result, in which the new exponent ν_a is larger than that in the normal regime, favours the node-link picture as far as SAW is concerned. In lower dimensions, especially in two dimensions, the node-link picture does not exist. In lower dimensions the fractal picture may be quite right for SAW. One should be careful to test discussions as above by simulations because the important lengths L and ξ_p diverge at the percolation threshold as one can see from (2) and (3). To test the crossover behaviour, one should take lattices large enough to see the crossover.

An apparent difficulty in our model in lower dimensions (Dasgupta *et al* 1978, Stanley 1977, Coniglio 1982) is that the node-link picture is not good for the backbone of the infinite cluster. In two dimensions the node-link picture cannot be used and we cannot apply our representation to SAW in two dimensions. That is why the exponents for the two dimensions are not displayed in table 1. Even in three dimensions, there may be several parallel paths between nodes and there may be several different physical lengths for the L of (3). Recently the nodes-links-blobs representation is believed to be more accurate for the backbone of the infinite cluster at the threshold (Stanley 1977, Coniglio 1982). This more advanced picture may be a good representation for SAW in lower dimensions. However, the numerical data for Kremer's simulation show that the node-link picture is quite good for the higher dimensions, especially near the upper critical dimension 6. There are not enough data to tell which representation is valid for SAW on the random lattice and in what dimension the new exponent ν_a in the anomalous regime becomes smaller than that of the normal regime. It is very important to study carefully the crossover phenomena by the simulation in each dimension between 2 and 6.

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